Decision Making Uncertain Environment - A Queuing Theory Approach

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Abstract**—** *Consumer behavior is one of the most uncertain phenomenons. Customer impatience is one of those uncertain phenomenon's which is a threat to any business. Customer impatience results in loss of customers and business. Stochastic modeling provides numerical measurement of necessary measures of performance in any business up-to a certain extent. In this paper a multi-server Markovian queuing system is developed with reverse balking and impatient customers. Reverse balking is a very new concept introduced in stochastic queuing models. While reneging is one of the most known phenomenon in queuing theory. Steady-state solution of the newly developed model is derived. Necessary measures of performance are obtained and numerical results are presented. Sensitivity analysis of the model is also performed. MATLAB and MS Excel are used as and when needed.*

Keywords— Reverse balking, customer impatience, retention of customers, stochastic modeling, queuing theory.

I. INTRODUCTION AND LITERATURE REVIEW

In this era of globalization and liberalization managing business has become a challenging task. Consumer behavior is one of the most uncertain characteristics of business environment. Customers have become more selective. Brand switching is more frequent. Due to higher level of expectations, customers get more impatient with a particular firm. Customer impatience has also become a burning problem in the corporate world. Queuing theory offers various stochastic models that can be used in various service systems facing customer impatience. By adopting and applying these stochastic models strategy making becomes highly effective. The premier work on customer impatience in queuing theory appeared in [Haight, 1957, 1959], [Anker & Gafarian, 1963a, 1963b], [Bareer, 1957] etc. Since then a number of papers have appeared on this concept (reneging and balking). In these models, reneging and balking is a function of system size/ queue length. Larger is the system size more is the reneging and similar is the case of balking. *But, when it comes to sensitive businesses like investment, selection of a food court, selection of a service station etc. more number of customers with a particular firm become the attracting (investing) factor for more investing customers*. Thus, the probability of joining in such a firm is high. Modeling such a system as a queuing system indicates that the probability of balking will be low when the system size is more and vice-versa, which is balking in the reverse sense (we call it *Reverse Balking*).

The concept of reverse balking is introduced by [Jain, et. al., 2014], they studied a single server Markovain queuing system with reverse balking. [Kumar et. al., 2014] further introduce notion of reverse reneging and applied it with reverse balking. [Kumar et. al., 2013, 2014] designed queuing systems for various environments and further optimized them for various parameters.

Finding impatience a threat to business firms employ various strategies to retain a reneging customer and they manage to do it with some probability. [Kumar, et. al., 2011] introduced the concept of retention of reneged customer in their work. They study a single –server queuing system with retention of reneged customers and balking. [Kumar, et. al., 2012] also study a multi-server queue with discouraged arrivals and retention of reneged customers. [Kumar, et. al., 2012, 2012a] further extend their work on single and multi-server feedback queues. Literature survey unfolds the need of the study due to following reasons.

Once a customer moves in to the system by looking at the large number of customers already present in the system, he may find the service unsatisfactory, as it is difficult for the firms to handle a huge chunk of customers at times. The customer starts experiencing delay and dissatisfaction in service. The customer becomes impatient due to this and considers leaving the system without completion of his service. This customer impatience can be termed as *Reneging* in queuing literature. [Som, 2014] developed a single-server queuing model by incorporating customer impatience and reverse balking. He also performed economic analysis of the model. Extending the work of the paper.

Owing to the practically valid aspects of above mentioned concepts, *sensitive businesses with customer impatience are formulated as queuing system in this paper*. Consider any life insurance company, where the purchase of policy refers to the arrival of a customer in the queuing system (insurance firms), the processed claim refers to as the departure from the queuing system, where the claim processing department is a multi-server and finite system capacity (the number of policies it can accommodate). The claims are processed in order of their arrival (i.e. the queue discipline is FCFS). We incorporate the reverse balking and reneging into this model. The model is based in Markovian assumptions.

We present steady-state analysis of the stochastic models as described above and derive important measures which help in the management of sensitive businesses like investment business. Numerical examples are provided for more clarity. Rest of the paper is structured as follows: in section 2 assumptions under which the model is developed are presented; section 3 deals with the mathematical formulation; in section 4 steady state solution is derived; section 5 deals with measures of performance; Numerical illustrations and sensitivity analysis of the model is performed in section 6; conclusions and future work are provided in section7.

II. MODEL ASSUMPTIONS

1. The arrival to a queuing system (insurance firm) occur, one by one in accordance with a Poisson process with mean rateλ. The inter-arrival times are independently, identically and exponentially distributed with parameterλ.
2. There is a multi-server and the policy claims are processed in parallel. The service times are independently, identically and exponentially distributed with parameter μ such as $\mu = n\mu$ for $n < c$. $\mu = c\mu$ for $n \geq c$.
3. The capacity of the system is finite, say *N*.
4. The policy claims are processed in order of their arrival, i.e. the queue discipline is First-come, First-served.
5. (a) When the system is empty, the customers balk (do not purchase policy) with probability and may purchase with probability p' (= 1 – q').
   (b) When there is at-least one customer in the system, the customers balk with a probability $1 - \frac{n}{N-1}$ and join the system with probability$\frac{n}{N-1}$. Such kind of balking is referred to as *reverse balking*.
6. The policy holders keeping their policies in force after some time, say T may get impatient due to certain reasons and decide to surrender their services before completion (the customer wait up-to certain time T and may leave the system before getting service due to impatience). The reneging times (T) are independently, identically and exponentially distributed with parameterξ.

III. STOCHASTIC MODEL FORMULATION

Differential difference equations of the model is given by:

$$\frac{dP_0(t)}{dt} = -\lambda p' P_0(t) + \mu P_1(t) \qquad ; n = 0 \quad (1)$$

$$\frac{dP_1(t)}{dt} = \lambda p' P_0(t) - \left\{\left(\frac{1}{N-1}\right)\lambda + \mu\right\} P_1(t) + (2\mu) P_2(t) \qquad ; n = 1 \quad (2)$$

$$\frac{dP_n(t)}{dt} = \left(\frac{n-1}{N-1}\right) \lambda P_{n-1}(t) - \left\{\left(\frac{n}{N-1}\right)\lambda + n\mu\right\} P_n(t) + \{(n+1)\mu\} P_{n+1}(t) \qquad 2 \leq n < c \quad (3)$$

$$\frac{dP_n(t)}{dt} = \left(\frac{n-1}{N-1}\right) \lambda P_{n-1}(t) - \left\{\left(\frac{n}{N-1}\right)\lambda + c\mu + (n-c)\,\xi\right\} P_n(t) + [c\mu + \{(n+1) - c\}\xi] P_{n+1}(t) \qquad n \geq c \quad (4)$$

$$\frac{dP_N(t)}{dt} = \lambda\, P_{N-1}(t) - [c\mu + (N-c)\xi]\, P_N\,(t)$$

; n = N (5)

IV. STEADY- STATE SOLUTION

In steady state $\lim_{t\to\infty} P_n(t) = P_n$, $\lim_{t\to\infty} P_n{}'(t) = 0$. Therefore the equations (1) to (5) become:

$$0 = -\lambda\, p^{'} P_0 + \mu\, P_1$$

; n =0 (6)

$$0 = \lambda\, p' P_0 - \left\{\left(\frac{1}{N-1}\right)\lambda + \mu\right\} P_1 + (2\mu)\, P_2$$

; n =1 (7)

$$0 = \left(\frac{n-1}{N-1}\right)\lambda\, P_{n-1} - \left\{\left(\frac{n}{N-1}\right)\lambda + n\mu\right\} P_n + \{(n+1)\mu\}\, P_{n+1}$$

$2 \le n < c$ (8)

$$0 = \left(\frac{n-1}{N-1}\right)\lambda\, P_{n-1} - \left\{\left(\frac{n}{N-1}\right)\lambda + c\mu + (n-c)\,\xi\right\} P_n + [c\mu + \{(n+1)-c\}\,\xi]\, P_{n+1}$$

$n \ge c$ (9)

$$\frac{dP_N(t)}{dt} = \lambda\, P_{N-1}(t) - [c\mu + (N-c)\xi]\, P_N\,(t)$$

; n = N (10)

Steady-state solution of the model is obtained by solving (6) – (10) iteratively. Probability of n customers in the system can be given by:

$$P_n = \begin{cases} \left[\dfrac{(n-1)!}{(N-1)^{n-1}} \displaystyle\prod_{r=1}^{n} \dfrac{\lambda}{r\mu}\right] p' P_0, n < c \\ \left[\dfrac{(n-1)!}{(N-1)^{n-1}} \displaystyle\prod_{s=c}^{n} \dfrac{\lambda}{c\mu + (s-c)\xi} \prod_{r=1}^{c-1} \dfrac{\lambda}{r\mu}\right] p' P_0, n \ge c \\ \left[\dfrac{(N-2)!}{(N-1)^{N-1}} \displaystyle\prod_{s=c}^{n} \dfrac{\lambda}{c\mu + (s-c)\xi} \prod_{r=1}^{c-1} \dfrac{\lambda}{r\mu}\right] p' P_0, n = N \end{cases}$$

Using the normalization condition $\sum_{n=1}^{N} P_n = 1$, we get

$$P_0 + \sum_{n=1}^{c-1} P_n + \sum_{n=c}^{N-1} P_n + P_N = 1$$

$$P_0 = \left\{1 + \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{r=1}^{n} \frac{\lambda}{r\mu}\right] p^{'} + \left[\frac{(n-1)!}{(N-1)^{n-1}} \prod_{s=c}^{n} \frac{\lambda}{c\mu + (s-c)\xi} \prod_{r=1}^{c-1} \frac{\lambda}{r\mu}\right] p^{'} + \left[\frac{(N-2)!}{(N-1)^{N-1}} \prod_{s=c}^{n} \frac{\lambda}{c\mu + (s-c)\xi} \prod_{r=1}^{c-1} \frac{\lambda}{r\mu}\right] p'\right\}^{-1}$$

V. MEASURES OF PERFORMANCE

5.1 Expected System Size

$$L_s = \sum_{n=1}^{N} nP_n$$

$$L_s = \sum_{n=1}^{c-1} nP_n + \sum_{n=c}^{N-1} nP_n + NP_N$$

$$L_s = \sum_{n=1}^{c-1} n\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{r=1}^{n}\frac{\lambda}{r\mu}\right]p'P_0 \; + \sum_{n=c}^{N-1} n\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{s=c}^{n}\frac{\lambda}{c\mu+(s-c)\xi}\prod_{r=1}^{c-1}\frac{\lambda}{r\mu}\right]p'P_0$$
$$+ N\left[\frac{(N-2)!}{(N-1)^{N-1}}\prod_{s=c}^{n}\frac{\lambda}{c\mu+(s-c)\xi}\prod_{r=1}^{c-1}\frac{\lambda}{r\mu}\right]p'P_0$$

5.2 Average rate of reneging

$$R_r = \sum_{n=c}^{N}(n-c)\xi\, P_n$$

$$R_r = \sum_{n=c}^{N-1}(n-c)\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{s=c}^{n}\frac{\lambda}{c\mu+(s-c)\xi}\prod_{r=1}^{c-1}\frac{\lambda}{r\mu}\right]\xi p'^{P_0} + (N$$
$$-c)\xi\left[\frac{(N-2)!}{(N-1)^{N-1}}\prod_{s=c}^{n}\frac{\lambda}{c\mu+(s-c)\xi}\prod_{r=1}^{c-1}\frac{\lambda}{r\mu}\right]\xi p'P_0$$

5.3 Average rate of reverse balking

$$R_b' = q'\lambda P_0 + \sum_{n=1}^{N-1}\left(1-\frac{n}{N-1}\right)\lambda\, P_n$$

$$R_b' = q'\lambda P_0 + \sum_{n=1}^{c-1}\left(1-\frac{n}{N-1}\right)\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{r=1}^{n}\frac{\lambda}{r\mu}\right]p'P_0 \; + \sum_{n=c}^{N-1}\left(1-\frac{n}{N-1}\right)\lambda\left[\frac{(n-1)!}{(N-1)^{n-1}}\prod_{s=c}^{n}\frac{\lambda}{c\mu+(s-c)\xi}\prod_{r=1}^{c-1}\frac{\lambda}{r\mu}\right]p'P_0$$

VI. NUMERICAL ILLUSTRATION

In table -1, numerical results of all measures of performance are presented. Numerical results are obtained for various rates of service.

Table.1:

λ =10, ξ =0.1, q′ =0.8, c=3, N =15

Rate of Service (μ)	Expected System Size (L_s)	Average Rate of Reneging (R_r)	Average Rate of Reverse Balking (R_b')
3.0	0.58730	0.00074	8.25796
3.1	0.56672	0.00060	8.25999
3.2	0.54789	0.00049	8.26106
3.3	0.53057	0.00040	8.26137
3.4	0.51455	0.00033	8.26110
3.5	0.49966	0.00028	8.26035
3.6	0.48576	0.00024	8.25923

3.7	0.47274	0.00020	8.25781
3.8	0.46051	0.00017	8.25616
3.9	0.44899	0.00015	8.25431
4.0	0.43811	0.00013	8.25232
4.1	0.42781	0.00011	8.25021
4.2	0.41804	0.00009	8.24801
4.3	0.40875	0.00008	8.24575
4.4	0.39991	0.00007	8.24343
4.5	0.39148	0.00006	8.24108
4.6	0.38343	0.00006	8.23870
4.7	0.37573	0.00005	8.23631
4.8	0.36836	0.00004	8.23392
4.9	0.36129	0.00004	8.23152
5.0	0.35451	0.00003	8.22914

An increasing rate of service ensures a large number of serviced customers leaving the system that leaves a negative impact on system size. This can be observed from table -1. Following figure shows change in system size with increasing rate of service.

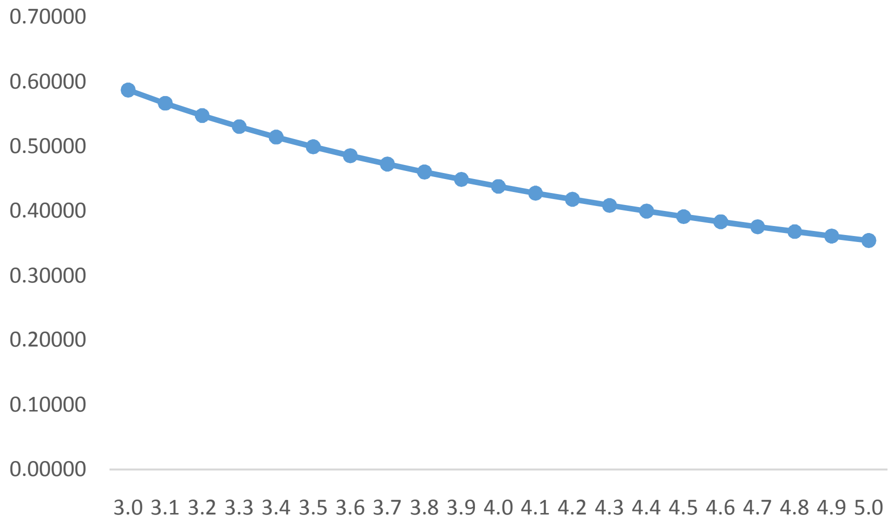

Fig.1: Ls Vs μ

6.1 Sensitivity Analysis

In this section sensitivity analysis of the model is presented. Variations in required measures of performance are observed with respective variable. Results are presented through graphs for better insight

Table.2:

$\mu = 3, \xi = 0.1, q' = 0.8, c = 3, N = 15$

Mean Arrival Rate (λ)	Expected System Size (L_s)	Average Rate of Reneging (R_r)	Average Rate of Reverse Balking (R_b')
5	0.29905	0.00001	4.10322
6	0.35450	0.00003	4.93749
7	0.41003	0.00008	5.77227
8	0.46650	0.00018	6.60563

9	0.52505	0.00038	7.43524
10	0.58730	0.00074	8.25796
11	0.65557	0.00138	9.06939
12	0.73319	0.00250	9.86309
13	0.82493	0.00436	10.62960
14	0.93730	0.00738	11.35519
15	1.07892	0.01209	12.02055
16	1.26051	0.01922	12.59966
17	1.49435	0.02960	13.05966
18	1.79298	0.04416	13.36238
19	2.16673	0.06371	13.46872
20	2.62057	0.08878	13.34601
21	3.15088	0.11936	12.97729
22	3.74367	0.15476	12.36948
23	4.37553	0.19362	11.55642
24	5.01751	0.23411	10.59444
25	5.64072	0.27431	9.55130

From table -2 it is clearly visible that, with increase in average arrival rate, expected system size increases. An increasing expected system size leads to high confidence of customers with the firm and rate of reverse balking decreases therefore. Due to this more and more arriving customers join the particular firm. The insight can be observed from graph below. On other hand rate of reneging increases gradually as increasing number creates a dense network due to high system size that leads to high level of impatience.

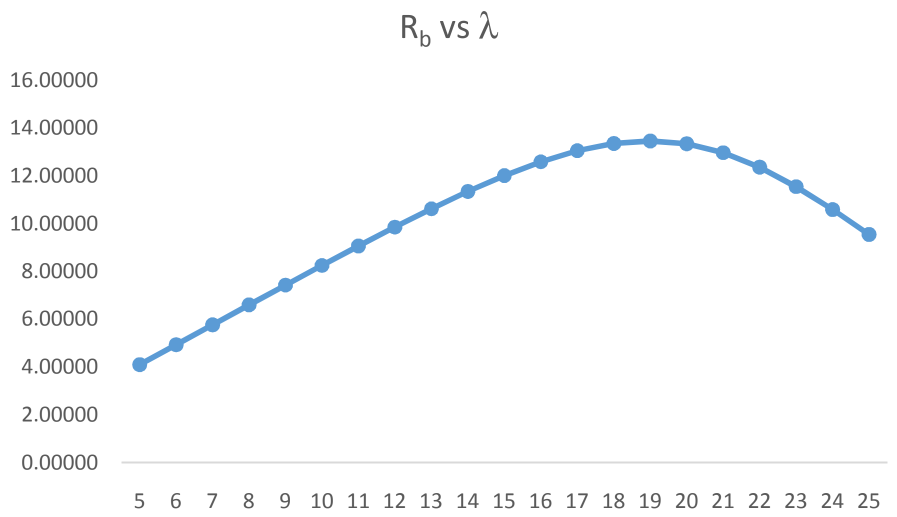

Fig.2: R_b vs λ

Figure -1 clearly states that more and more arrivals cause an increase in system size due to which rate of reverse balking decreases.

Table.3:
μ =3, λ =2, q′ =0.2, c=3, N =15

Rate of Reneging (ξ)	Expected System Size (L_s)	Rate of Reneging (R_r)
0.05	0.354518	0.000018
0.06	0.354516	0.000021
0.07	0.354515	0.000024
0.08	0.354513	0.000028
0.09	0.354512	0.000031
0.1	0.354510	0.000035
0.11	0.354509	0.000038
0.12	0.354507	0.000041
0.13	0.354505	0.000045
0.14	0.354504	0.000048
0.15	0.354502	0.000052

From table -3, it can be observe that increasing rate of reneging causes decrease in expected system size and increase in average rate of reneging. This is because increasing rate of reneging states that more and more customers are moving out of the system without completing their service.

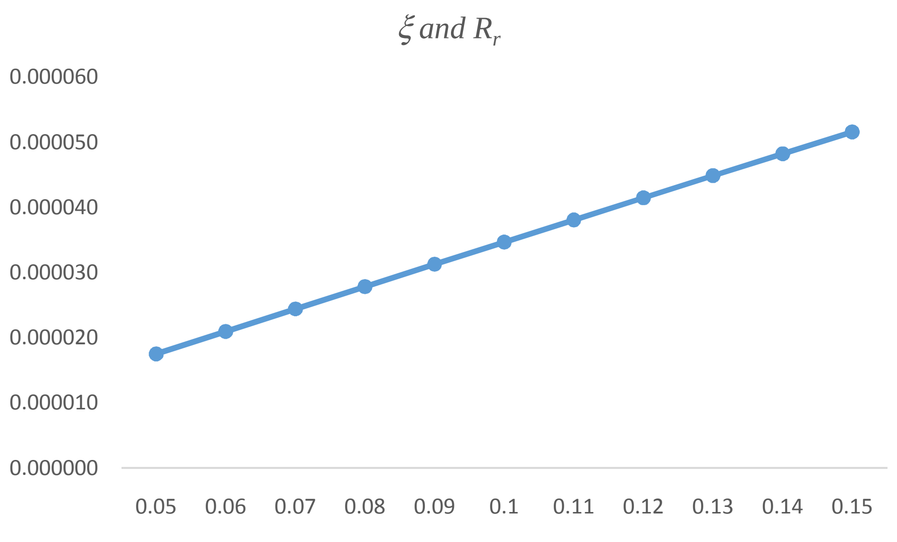

Fig.3: ξ vs R_r

Figure -3 represents increase in average rate of reneging with increase in reneging rate that is obvious.

Table.4:
ξ =0.2, μ =3, λ =10, c=3, N =15

Probability of Reverse Balking (q')	Expected System Size (L_s)	Average Rate of Reverse Balking (R_b')
0.1	1.01918	0.00128

0.2	0.99310	0.00125
0.3	0.96146	0.00121
0.4	0.92229	0.00116
0.5	0.87253	0.00109
0.6	0.80719	0.00101
0.7	0.71763	0.00090
0.8	0.58730	0.00074
0.9	0.38018	0.00048
1.0	0.00000	0.00000

It can be observed from table -5, that with increase in probability of reverse balking when there were no customers in the system expected system size reduces and at q' =1 (probability that an arriving customer does not join the system) expected system size drops to zero. And R_b' = 10, states that all arriving customers reverse balked.

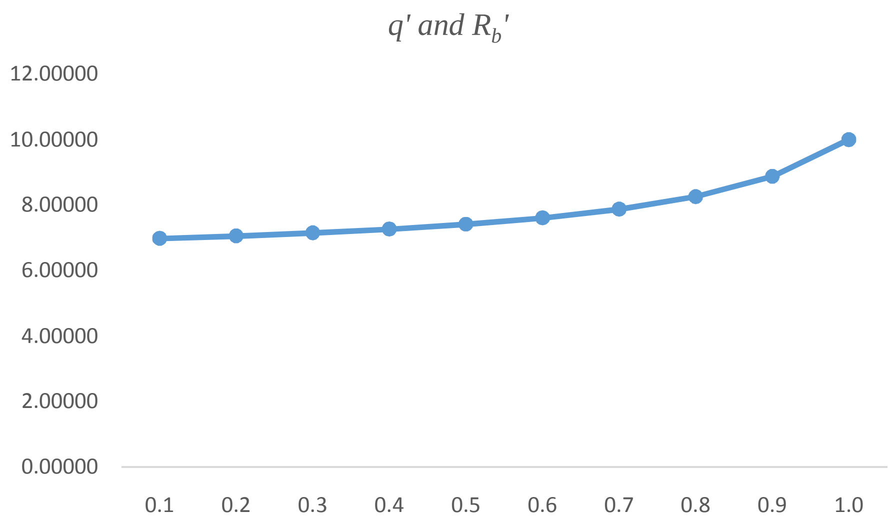

Fig.4: q' vs R_b'

Figure -4, represents increasing rate of reverse balking w.r.t. increase in probability of reverse balking.

VII. CONCLUSION

In this paper a multi-server Mrkovian queuing system with reverse balking and reneging of customers is developed. Steady-state solution of the model is derived. Necessary measures of performance are obtained. Numerical results are obtained by writing and algorithm in MS Excel and MATLAB. Sensitivity analysis of the model is also performed. Measures of performance with relevant variables are studied.

The results are of immense use for making growth strategies. The model mentioned above can be tailor-made as per need and want for the firms operating in uncertain business environment. In future cost-profit analysis of the model can be presented with optimization.

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